NEUROMORPHIC SAMPLING

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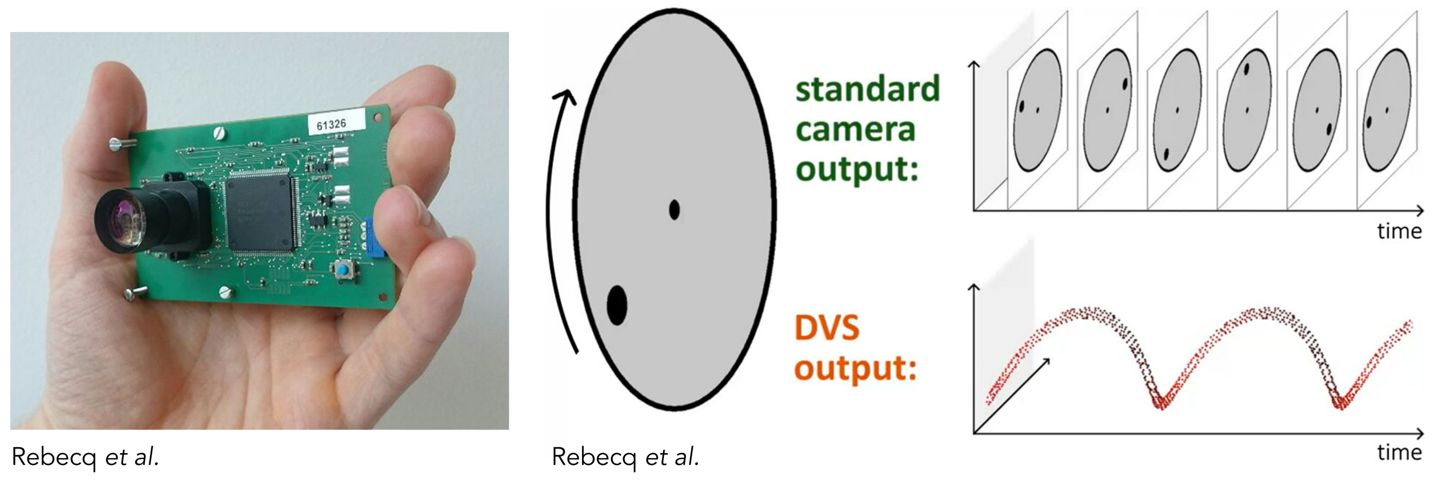
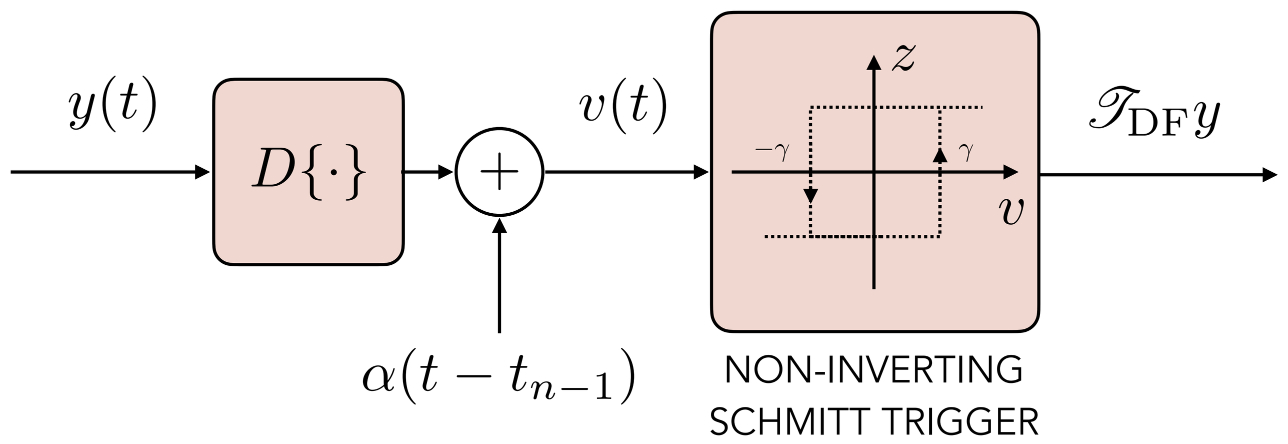
1. INTRODUCTION

- Conventional sampling of continuous-time signals relies on uniform sampling of the amplitude.
- Dynamic Vision Sensors (DVS) represent scenes as a sequence of events



- *Definition:* A time-encoding machine with an event operator $\mathscr{E}:\mathbb{R}^{\mathbb{R}}\to\mathbb{R}^{\mathbb{R}}$ and references $\{r_n\in\mathbb{R}^{\mathbb{R}}\}_{n\in\mathbb{Z}}$ is a map $\mathscr{T}:\mathbb{R}^{\mathbb{R}}\to\mathbb{R}^{\mathbb{Z}}$ such that $\mathbb{R}^{\mathbb{R}}\ni y\mapsto\mathscr{T}y$, with
  - $\mathscr{T}y=\{t_i\in\mathbb{R}\mid t_i>t_j,\ \forall i>j, i\in\mathbb{Z}\}$,
  - $\lim_{n\to\pm\infty} t_n=\pm\infty$, and
  - $(\mathscr{E}y)(t_n)=r_n(t_n), \forall t_n\in\mathscr{T}y$.

2. DIFFERENTIATE-AND-FIRE TIME-ENCODING MACHINE



- *Lemma:* Let $y\in C^1(\mathbb{R})$ be the input to the DF-TEM. The output $\mathscr{T}_{\text{DF}}y=\{t_n\}_{n\in\mathbb{Z}}$ satisfies

$$(Dy)(t_n)=(-1)^{n+1}\left(\gamma-\alpha(t_n-t_{n-1})\right),\ \forall t_n\in\mathscr{T}_{\text{DF}}y.$$

- *Corollary:* Let $y\in C^1(\mathbb{R})$ with $\|Dy\|_\infty\le\beta$ be the input to the DF-TEM. The output $\mathscr{T}_{\text{DF}}y=\{t_n\}_{n\in\mathbb{Z}}$ satisfies

$$d(\mathscr{T}_{\text{DF}}y)\doteq\sup_{n\in\mathbb{Z}}|t_n-t_{n-1}|\le\frac{\gamma+\beta}{\alpha}.$$

3. TIME-ENCODING OF SIGNALS IN SHIFT-INVARIANT SPACES

- Consider signals in the integer shift-invariant space $V(\varphi)$:

$$y(t)=\sum_{k\in\mathbb{Z}}c_k\varphi(t-k)$$

- $\{\varphi(\cdot-k)\}_{k\in\mathbb{Z}}$ forms a Riesz basis for $V(\varphi)$ and the coefficient sequence $\tilde{\mathbf{c}}=\{c_k\}_{k\in\mathbb{Z}}$ defines the signal.

The reconstruction problem: Given $\mathscr{T}_{\text{DF}}y$, find y.

▪ METHOD OF ALTERNATING PROJECTIONS

- Consider the linear operator $\mathcal{V}:\mathbb{R}^{\mathbb{Z}}\to\mathbb{R}^{\mathbb{R}}$ defined by

$$\mathcal{V}x(t)=\sum_{n\in\mathbb{Z}}x(t_n)\mathbb{1}_{[s_n,s_{n+1}[}(t).$$

- *Lemma:* Let $\varphi, D\varphi, D^2\varphi\in L^2(\mathbb{R})$. Let the operator $\mathcal{V}$ be defined with the set $\{t_n\}_{n\in\mathbb{Z}}$ having increasing entries and bounded density $T=d(\{t_n\}_{n\in\mathbb{Z}})<\infty$. Then, $\forall y\in V(\varphi)$,

$$\|Dy-\mathcal{V}Dy\|^2_{L^2(\mathbb{R})}\le\underbrace{\left(\frac{T}{\pi}\sup_{\omega\in[0,2\pi[}\frac{G_{D^2\varphi}(\omega)}{G_{D\varphi}(\omega)}\right)^2}_{\eta}\|Dy\|^2_{L^2(\mathbb{R})},$$

where $G_\varphi(\omega)=\left(\sum_{k\in\mathbb{Z}}|\hat{\varphi}(\omega+2\pi k)|^2\right)^{1/2}$.

- MAP iterations: $\Pi=\Pi_{V(D\varphi)}$, $x_1=\Pi\mathcal{V}Dy$ and

$$x_{\ell+1}=x_1+(\text{Id}-\Pi\mathcal{V})x_\ell.$$

We have $\|Dy-x_k\|^2_{L^2(\mathbb{R})}\le\eta^k\|Dy\|^2_{L^2(\mathbb{R})}\underset{k\to+\infty}{\longrightarrow}0.$

▪ BANDLIMITED SIGNALS

- Consider bandlimited signals $y\in B([-\pi,\pi])$. The derivative signal $Dy\in B([-\pi,\pi])\cap V(D\text{sinc})$, i.e.,

$$Dy(t)=\sum_{k\in\mathbb{Z}}y(k)\ D\text{sinc}(t-k)=\sum_{k\in\mathbb{Z}}Dy(k)\ \text{sinc}(t-k).$$

- Samples of the derivative can be obtained using MAP.
- Samples $y(k)$ can be obtained from derivative samples

$$y(k)=\sum_{m\in\mathbb{Z}}Dy(m)\frac{\text{Si}\left((k-m)\pi\right)}{\pi},$$

where $\text{Si}(t)=\int_0^t\frac{\sin(u)}{u}du$ is the Sine integral function.

▪ GENERATOR KERNELS WITH COMPACT SUPPORT

- Consider the signals generated by finite sequences $\mathbf{c}=[c_0\ c_1\ \cdots\ c_{K-1}]^{\mathsf{T}}$ and generator kernels with $\text{supp}(\varphi)<K$. $L\ge K$ time-instants construct an invertible linear system of equations $\mathbf{y}=\mathbf{Mc}$, where $\mathbf{y}=[Dy(t_1)\ Dy(t_2)\ \cdots\ Dy(t_L)]^{\mathsf{T}}$, and $[M]_{i,j}=D\varphi(t_i-j)$, i.e., $\mathbf{c}$ can be uniquely recovered.

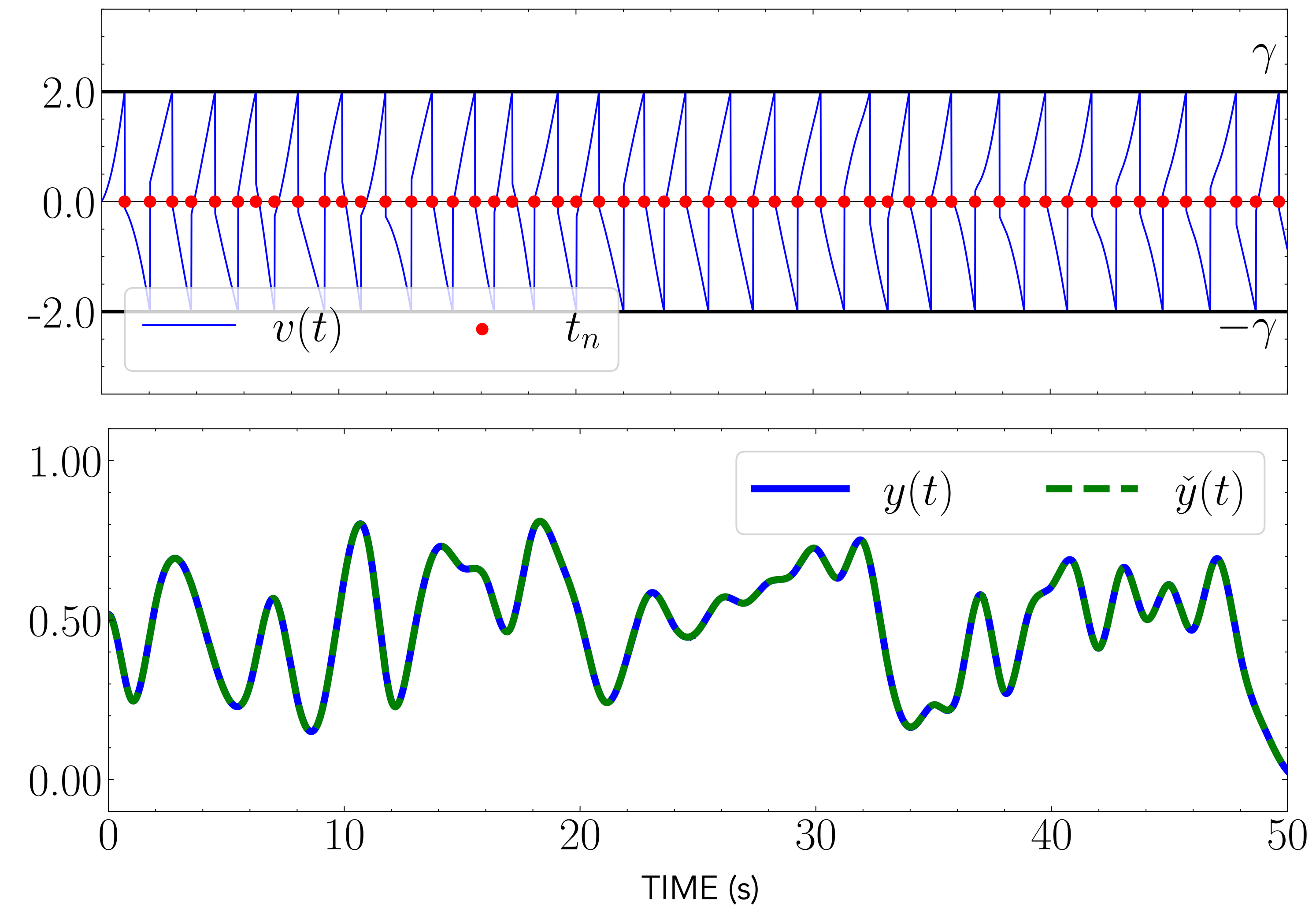


Figure: [Top] Input to the Schmitt trigger and output of the DF-TEM; [Bottom] Input to the DF-TEM in the cubic B-spline integer-shift-invariant space and its reconstruction.

4. KEY REFERENCES

- Gallego *et al.*, "Event-based vision: A survey." arXiv preprint arXiv:1904.08405, 2019.
- Lazar, "Time encoding with an integrate-and-fire neuron with a refractory period," Neurocomputing, 2004.
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